Mathematics – 101

Semester – I

December – 2011

[Time : 3 Hours] [Max. Marks 70]

Instrucitons : (1) Figure to the right side indicate full marks of question.

(2) Symbols are usual.

1. (a) State and prove Leibnitz Theorem. [7]

OR

Let $\sum_{i=1}^{\infty} a_i$ be infinite series of positive terms

and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = l$ then. P. T.

(1) If $l < 1$ then $\sum_{i=1}^{\infty} a_i$ is convergent.

(2) If $l > 1$ then Σa_i is divergent.

(b) If y = cos (ax + b); (a, b const.) then prove [7]

that $y_n = a^n \cos\left(ax + b\frac{n\pi}{2}\right)$; $n \in N$.

OR

Discuss the convergence of the following series :

(1) $\Sigma \frac{4'' \pm 5''}{20''}$ (2) $\frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \ldots..$

2. (a) State and prove Rolle's Theorem [7]

OR

State L' Hospital's rule and find

$\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan^2 x}$

(b) State Taylor's expansion theorem and expand $\sqrt{x}$ in powers of $(x - 4)$. **[7]**

OR

State and prove L' Hospital's second rule.

3. (a) Define Hermitian and skew – Hermitian matrix. **[7]**

Express matrix $A = \begin{bmatrix} 2+i & -1-i & 3 \\ 1+i & 5 & 4-3i \\ -2i & 1+3i & -2-7i \end{bmatrix}$ as a sum of Hermitian and skew Hermitian matrix.

OR

Define transpose of a matrix. Prove that $(AB)^T = B^T A^T$ for matrix A of order $m \times n$ and matrix B of order $n \times p$.

(b) Verify $A\,(\text{adj } A) = (\text{adj } A) . A = |A| . I_3$ **[7]**

For $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}$. Also find A^{-1}.

OR

Find the rank of a matrix

$$A = \begin{bmatrix} 3 & 2 & 0 & -1 \\ 1 & -1 & 2 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

4. (a) State and prove Cayley's Hamilton Theorem. **[7]**

OR

Verify Cayley Hamilton theorem for matrix

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Also find A^{-1}

(b) If λ is an eigen value matrix $A = [a_{ij}]_n$ **[7]**
then show that,

(i) $\frac{1}{\lambda}$ is the eigen value of A^{-1}.

(ii) $\frac{|A|}{\lambda}$ is the eigen value of adj A.

(iii) λ^3 is the eigen value of A^3.

OR

Solve $5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$; $7x + 2y + 11z = 5$ using Cramer's rule.

5. **Answer the following in short.** **[14]**

(i) Write expansion of sin x in terms of x.

(ii) If $y = \log(3x - 4)$ then what is y_n ?

(iii) Define Convergence of a series.

(iv) Evaluate : $\lim\limits_{x \to \infty} \frac{3^x - 2^x}{x}$

(v) Define row rank and column rank of a matrix.

(vi) Give a function which is not differentiabel but continuous.

(vii) For which value of p the series $\Sigma \frac{1}{n^p}$ is convergent ?

(viii) If $A = \begin{bmatrix} 5 & x \\ 7 & 0 \end{bmatrix}$ then solve : $A = A'$.

(ix) Write necessary and sufficient condition for a square matrix possess inverse.

(x) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ then what is A^{-1} ?

(xi) If one eiegn Value of A is –2 what will be eigen value of A^2 ?

(xii) Define Diagonal Matrix.

(xiii) How many minors does a 3×4 matrix have ?

(xiv) What is the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Gujarat University

Sem - I

December - 2012

Mathematics

Paper - 101

[Time : 3 Hours] [Max. Marks : 70]

Instructions :
(1) There are 5 questions.
(2) Fifth question is objective type.
(3) All questions are compulsory.

1. (a) If $y = e^{ax} \cos(bx + c)$ a, b, c $\in$ R, then prove that $y_n = r^n e^{ax} \cos(bx + c + n\theta)$ where $a = r\cos\theta$ $b = r\sin\theta$; $\theta = \tan^{-1} b/a$.

OR

If $y = \log(ax + b)$ where $ax + b > o$, a, b, c$\in$R,

then prove that $y_n = \dfrac{(-1)^{n-1}(n-1)!\, a^n}{(ax+b)^n}$

(b) State and prove Cauchy's root test for the convergence of the infinite positive series.

OR

Discuss the convergence of the following series :

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^3+1}$

(2) $\displaystyle\sum \left(1 - \frac{1}{2n}\right)^{n^2}$

2. (a) State and prove the Lagrange's mean value theorem.

OR

State and prove L'Hospital's First rule.

(b) State the Maclaurin's theorem. Using this obtain cos x in the powers of x.

OR

Find the value :

(1) $\lim_{x \to 0} (\sec^2 x) \cot^2 x$

(2) $\lim_{x \to 1} \left[\frac{1}{\log x} - \frac{1}{x-1}\right]$

3. (a) Define adjoint of a matrix. For a square matrix of order n. Prove that A -(adj A) = (adj A) A = $|A|\ I_n$

OR

Define transpose of a matrix. Prove that $(AB)^T = B^T A^T$ for matrix of A of order m x n and matrix B of order n x p.

(b) Express the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 4 & 0 & 2 \\ 1 & 1 & -2 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix.

OR

For matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 2 & 2 \\ -1 & 1 \end{bmatrix}$ verify the result $(AB)^T = B^T A^T$.

4. (a) Prove that every square matrix is satisfied it's characteristic equation.

OR

If λ is an eigen value of matrix $A = (a_{ij})_n$ then show that

(1) $\frac{1}{\lambda}$ is the eigen value of A^{-1}

(2) $\frac{|A|}{\lambda}$ is the eigen value of adj A.

(b) Find Eigen value and Eigen vector of the given matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

OR

Solve' $3x - y + z = 4$; $x + y - z = 0$; $2x - y + z = 3$ using Cramer's rule.

5. Answer the following in short :

(1) State the Leibnitz theorem.

(2) If $y = \frac{1}{\sec(1-x)}$ then what is y_n ?

(3) If $y = 2^{3x} + 1$ then find y_n

(4) When alternative series is convergent ?

(5) For what value of P : $\Sigma \frac{1}{np}$ is divergent ?

(6) State the second L'. Hospital rule.

(7) If $A = \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$ then what is characteristic value ?

(8) Find the value of $\lim_{x \to 1} x^{\frac{1}{1-x}}$

(9) What do you mean by In-determinate form ?

(10) For a system of linear equation when

(i) Solution does not exists ?

(ii) When it has unique solution ?

(11) State and prove De'Alembert's Ratio test.

(12) If A is symmetric matrix then what about $A + A^T$ and $A - A^T$?

(13) Define rank of a matrix.

(14) With illustrate, define Hermition matrix.

★★★★★

Gujarat University

B.Sc. (Sem - I)

December - 2013

CC-3 : Mathematics

Paper - 101

[Time : 3 Hours] [Max. Marks : 70]

Instructions : (1) There are **five** questions.

(2) **Fifth** question is objective.

1\. (a) If $y = e^{ax} \cdot \sin(bx + c)$ a, b, c $\in$ R then prove that

$$y_n = (a^2 + b^2)^{n/2} \cdot e^{ax} \sin\left(bx + c + n \tan^{-1}\frac{b}{a}\right).$$

where $\tan\theta = \dfrac{b}{a}$. 7

OR

If $y = (ax + b)^m$; $ax + b \in R$ $a \neq 0$ and b is constant then find out y_n for $n \in N$.

(b) State and prove De' Alembert Ratio Test. 7

OR

Discuss the convergence for the following series :

(i) $$\frac{x}{2.3} + \frac{x^2}{3.4} + \frac{x^3}{4.5} + \text{———}$$

(ii) $$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \text{———} + \frac{x^n}{n^2 + 1} + \text{———}.$$

2. (a) State and prove Cauchy's mean value theorem. 7

OR

State and prove L'Hospital's second rule.

(b) State the Maclaurin's theorem. Using this obtain 'sin x' in the powers of x. 7

OR

Prove that :

(i) $\lim_{x \to 0^+} \left(\frac{1}{x}\right)^{\tan x} = 1 \; ; x > 0$

(ii) $\frac{x}{1+x^2} < \tan^{-1} x < x$ and hence show that

$$\pi \in \left(\frac{3\sqrt{3}}{4}, 3\sqrt{3}\right).$$

3. (a) Define Skew symmetric Matrix and Skew-Hermitian matrix. 3

If A and B are symmetric matrices of the same order, then prove that AB–BA is a skew-symmetric matrix. Also prove that AB–BA is a skew - Hermitian, if A and B are Hermitian matrices of the same order. 4

OR

Define Transpose of a matrix and Diagonal matrix. 3

Prove that $(AB)^T = B^T A^T$ for matrix A of order $m \times n$ and matrix B of order $n \times p$. 4

(b) Define : Conjugate matrix and conjugate transpose matrix. For given matrix. 7

$$A = \begin{bmatrix} 1+i & -2i \\ 3-i & 4+i \\ 1 & 3-2i \end{bmatrix}_{3 \times 2}$$ Prove that $A^* = \overline{\left(A^T\right)}$.

OR

Find A^{-1} of matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 1 & 3 & 1 \end{bmatrix}_{3 \times 3}$

4. (a) If λ is an eigen value of matrix $A = [a_{ij}]_n$ then show that ——— .

(i) λ^{-1} is the eigen value of A^{-1}. 7

(ii) $\dfrac{|A|}{\lambda}$ is the eigen value of adj A.

OR

Find Eigen value and Eigen vectors of the given matrix

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}.$$

(b) Verify Caley - Hamilton theorem for the given matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

Also using this theorem find A^{-1}. 7

OR

Solve the following equations by Crammer's rule :

$x + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y - z = 0$.

5. **Answer the following questions in short :** 14

(1) If $y = 5^{2x-1}$, then what is y_n ?

(2) If $y = (ax + b)^m$, $ax + b \in R$ and $a \neq 0$, b are constants then what is y_n for $n = m$ and $n > m$?

(3) For what value of p; $\Sigma \dfrac{1}{n^p}$ is convergent and divergent.

(4) When alternative series is convergent ?

(5) Let function 'f', defined in the interval [a,b]. When you say that it is strictly increasing ?

(6) Show that other forms of Lagrange's mean value theorem.

(7) What do you mean by Indeterminant form ?

(8) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, then which type of matrix AB ?

(9) Let A be a square matrix. Then show that $A + A^*$ is Hermitian.

(10) If A is symmetric matrix then what about $A + A^T$ and $A - A^T$?

(11) Define : Adjoint Matrix.

(12) Define : Consistent and Inconsistent system.

(13) Fill in the blanks : adj A . A = ...

(14) What is characteristic equation of the matrix A ?

★★★★★

Gujarat University

B.Sc. (Sem - I)

December - 2014

MAT - 101 : Mathematics

(Calculus and Matrix Algebra)

[Time : 3 Hours] [Max. Marks : 70]

Instructions : (1) There are five questions.

(2) Fifth question is objective.

(3) All questions carry equal marks.

1. (A) State and prove Leibnitz's theorem. (06)

OR

(A) State and prove De' Alembert ratio test.

(B) (1) Find $\left\{x \log\left(\frac{x-1}{x+1}\right)\right\}_n$ (08)

(2) If $y = e^{m \sin^{-1} x}$ then prove that

$$(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} = (n^2+m^2)\, y_n$$

OR

Discuss the Convergence of the following series :

(1) $\Sigma[(n^3+1)^{1/3}-n]$

(2) $\Sigma \dfrac{n}{n^2+1} x^n$

2. (A) State and prove Lagrange mean value theorem. (06)

OR

(A) State Maclaurin's theorem. Using this obtain $\sin x$ in the powers of x.

(B) (1) If $3a - 4b + 6c - 12d = 0$, then show that one root of cubic equation $ax^3 + bx^2 + cx + d = 0$ lies between -1 and 0. (08)

(2) Verify Cauchy's mean value theorem for the functions $f(x) = \sqrt{x}$ and $g(x) = 2x + 1$ in the interval $[1, 4]$, If possible, then find 'c'.

OR

Evaluate limit.

(1) $\lim\limits_{x \to 0} (\sec^2 x)^{\cot^2 x}$

(2) $\lim\limits_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\cot^2 x}\right]$

3. (A) For matrix A of order $m \times n$ and matrix B of order $n \times p$, prove that $(AB)^T = B^T A^T$. (06)

OR

(A) For a square matrix A of order n, Prove that $A\,(\mathrm{adj}A) = (\mathrm{adj}A)\,A = |A|\,I_n$.

(B) (1) Find the rank of matrix $A = \begin{bmatrix} 3 & 2 & 0 & 1 \\ 1 & -1 & 2 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}$ (08)

(2) Express the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & -7 & 8 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix.

OR

(1) Find A^{-1} of a matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$.

(2) Verify A * A is a Hermitian matrix for a matrix

$$A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}.$$

4. **(A)** State and prove Cayley - Hamilton theorem. **(06)**

OR

(A) If λ is an Eigen value of matrix $A = [a_{ij}]_n$ then show that

(1) $\frac{1}{\lambda}$ is the Eigen value of A^{-1}.

(2) $\frac{|A|}{\lambda}$ is the Eigen value of *adj* A.

(B) (1) Discuss the consistency of the following system of equations. **(08)**

$x+2y+z=2,\ 2x+4y+3z=3,\ 3x+6y+5z=4$

(2) Find the Eigen values and Eigen vectors of

$$\text{matrix } A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

OR

(1) Using Cayley - Hamilton theorem find the inverse matrix of a matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

(2) For which values of λ and μ, the following system of equations has (i) no solution (ii) unique solution.

$x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu.$

5. Give answer in short. (14)

(1) If $y = (3x + 4)^5$ then find y_5.

(2) If $y = \dfrac{1}{cosec(1-2x)}$ then find y_n.

(3) Discuss the convergence of $\Sigma \dfrac{1}{n^{3/2}}$.

(4) When alternative series is convergent ?

(5) Can we apply Rolle's Theorem for funtion $f(x) = |x|,\ x \in [-1, 1]$.

(6) Write the expression for $cos\ x$ in terms of x.

(7) Show that the funtion $f(x) = x^3 + 1$, $x \in R$ increasing.

(8) What do you mean by indeterminant form ?

(9) Define : Lower triangular matrix with illustration.

(10) If A is skew - symmetric, what about $A - A^T$?

(11) Write the condition for the existence of inverse of a square matrix.

(12) Show that $A - A^*$ is skew - Hermitian for a square matrix A.

(13) Write the Eigen values of any diagonal matrix.

(14) Define : Consistent system.

★★★★★

Gujarat University

B.Sc. (Sem. I) Examination

January – 2016

CC - 3 – Paper - 101 : Mathematics

(Calculus and Matrix Algebra) (Theory)

[Time : 3 Hours] **[Max. Marks : 70]**

Instructions :

(1) There are **five** questions in this question paper.

(2) Fifth question is short answer type.

(3) All questions are compulsory.

(4) Symbols are usual.

(5) All questions carry **14** marks.

(6) The right side figure indicate marks of questions.

---

1. **(a)** If $y = e^{ax} \sin(bx + c)$, then prove that $y_n = r^n e^{ax} \sin(bx + c + n\alpha)$; where $a \neq 0$, $b \neq 0$, $c \in R$; $n \in N$ and $a = r \cos \alpha$; $b = r \sin \alpha$. **6**

OR

State and prove Cauchy's root test for the convergence of the infinite positive series.

(b) **Answer the following questions :** **8**

(1) If $y = x^3 \cdot \log x$, then find y_n.

(2) If $y = \cos^{-1} x$; $x \in (-1, 1)$, then prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$$

OR

Discuss the convergence for the following series :

(1) $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \frac{1}{11.14} + \ldots..$

(2) $\Sigma \frac{x^n}{n^2 + 1}$

2. (a) State and prove the Cauchy's mean value theorem. 6

OR

State and prove L' Pittal's Second Rule.

(b) **Answer the following questions :** 8

(1) State Taylor's expansion theorem and using this expand sin x in power of $\left(x - \frac{\pi}{2}\right)$.

(2) Prove that,

$$\frac{x}{1 + x^2} < \tan^{-1}x < x; \text{ where } 0 < x.$$

OR

Answer the following questions :

(1) Verify the Roll's mean value theorem for the function $f(x) = x^2 - 2x + 3$, $x \in [0, 2]$ and find $C \in (0, 2)$.

(2) Evaluate : $\lim_{x \to 0} \dfrac{\tan x - \sin x}{x^3}$.

3. (a) Define Hermitian and Skew-Hermitian matrices. 2

Express matrix $A = \begin{bmatrix} 2+i & -1-i & 3 \\ 1+i & 5 & 4-3i \\ -2i & 1+3i & -2-7i \end{bmatrix}$

as a sum of Hermitian and Skew-Hermitian matrices. 4

OR

For a square matrix A of order n, prove that $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A| \, I_n$. 4

Verify $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A| \, I_2$ for a matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$. 2

(b) **Answer the following questions :** 8

(1) Express the matrix $A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices.

(2) Find the rank of a matrix

$$A = \begin{bmatrix} 4 & 3 & 0 & -2 \\ 3 & 4 & -1 & -3 \\ -7 & -7 & 1 & 5 \end{bmatrix}$$

OR

Answer the following questions :

(1) For matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 2 & 2 \\ -1 & 1 \end{bmatrix}$

verify the result $(AB)^T = B^T A^T$.

(2) Find C^{-1} of square matrix $C = \begin{bmatrix} 2 & 3 & 6 \\ 1 & 2 & -1 \\ -4 & 0 & 7 \end{bmatrix}$.

4. **(a)** Verify Caley-Hamilton theorem for the given

matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Also using this

theorem find A^{-1}. **6**

OR

If λ ($\lambda \neq 0$) is an Eigen value of an invertible matrix $A = (a_{ij})_n$ then show that

(1) $\frac{1}{\lambda}$ is the eigen value of A^{-1}.

(2) $\frac{|A|}{\lambda}$ is the given value of adj A.

(b) **Answer the following questions :** 8

(1) Find the eigen value and eigen vector corresponding to any one eigen value of the square matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(2) Find the characteristic equation for matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Also find the matrix represented by the matrix polynomial $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

OR

Answer the following questions :

(1) Solve the equations $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ using Cramer's rule.

(2) Prove that the equations $x - 3y + z = -2$, $2x + y - z = 6$, $x + 2y + 2z = 2$ are consistent.

5. **Answer the following questions in short (any seven) :** 14

(1) If $y = \dfrac{1}{2x + 4}$ then find the value $y_6(1)$.

(2) Find the radius of convergence of the power series $\sum \left(\sqrt[n]{n} - \dfrac{1}{2} \right)^n x^n$.

(3) Write the expansion of $\log_e (1 + x)$ in terms of x.

(4) Evaluate : $\lim_{x \to 0} \frac{3^x - 2^x}{x}$.

(5) Define the transpose matrix with illustration.

(6) If $A = \begin{bmatrix} 0 & 11 \\ x & 0 \end{bmatrix}$ is skew-symmetric matrix then find the value of x.

(7) Find A^{-1} for the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

(8) If one eigen value of a square matrix A is (–3), what will be the eigen value of A^2 and A^3 ?

(9) Write the eigen value of matrix $A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$.

* * *