Seat No. : _____

LD-111

April-2014

B.Sc. Sem.-VI

CC-307 : Physics

(Mathematical Physics, Classical Mechanics and Quantum Mechanics)

Time : 3 Hours] **[Max. Marks : 70**

સૂચના : (1) બધા જ પ્રશ્નોના ગુણ સરખા છે.

Instructions : **All** questions carry equal marks.

(2) સંજ્ઞાઓ તેમના પ્રચલિત અર્થમાં છે.

Symbols have their usual meanings.

1. (અ) જો 'v' પૂર્ણાંક ન હોય તો દર્શાવો કે $W[Jv(x), J_{-v(x)}] = \frac{-2\sin \pi v}{\pi x}$ 7

Show that $W[Jv(x), J_{-v(x)}] = \frac{-2\sin \pi v}{\pi x}$ if 'v' is not an integer.

અથવા/OR

v = n પૂર્ણાંક માટે સાબિત કરો કે :

For v = n an integer, show that :

(1) $J_{v-1}(x) + J_{v+1}(x) = \frac{2v}{x} J_v(x)$

(2) $J_{v-1}(x) - J_{v+1}(x) = 2J'_v(x)$

(બ) સાબિત કરો કે $J_m(x) = \frac{1}{\pi}\int_0^n \cos(m\theta - x\sin\theta)d\theta$ 7

Prove that $J_m(x) = \frac{1}{\pi}\int_0^n \cos(m\theta - x\sin\theta)d\theta$

અથવા/OR

દર્શાવો કે :

Show that :

(1) $x P'_l(x) - l P_l(x) = P'_{l-1}(x)$

(2) $x P'_{l-1}(x) + l P_{l-1}(x) = P'_l(x)$

2

2. (અ) જીઓડેસીક્સ સમજાવો. ગોલીય સપાટીના જીઓડેસીક્સ ગુરૂ વર્તુળો છે. તેમ દર્શાવો. 7

Explain geodesic. Show that the geodesic of spherical surface are great circles.

અથવા/OR

લઘુત્તમ સમયનો કોયડો વર્ણવો. આ કોયડો કેવી રીતે ઉકેલાય છે તે સવિસ્તર સમજાવો.

Describe shortest time problem. How this problem can be solved ? Explain in detail.

(બ) વીજયાંત્રિક સરખામણી (electromechanical analogies)ને આધારે LCR શ્રેણી પરિપથ અને LCR સમાંતર પરિપથ માટે લાગ્રાન્જીયન મેળવો. 7

Obtain Lagrangian for series LCR and parallel LCR electric circuit on the basis of electro-mechanical analogies.

અથવા/OR

l ત્રિજ્યા ધરાવતા લીસા ગોલક (smooth sphere) પર ગુરૂત્વાકર્ષણની અસર નીચે m દ્રવ્યમાન ધરાવતો કણ ગતિ કરે છે. લાગ્રાન્જેના અનિર્ધારીત ગુણાંકની મદદથી ગોલકની સપાટી પરથી કણ માટેનો "flies off" ખૂણો શોધો (સપાટીનું ઘર્ષણ અવગણો).

A particle of mass m is moving under the action of gravity on the surface of smooth sphere of radius *l*. Apply Lagrangian method of undetermined multiplier to find out the angle at which the particle "flies out" of the surface. (neglect surface friction)

3. (અ) હાઈડ્રોજન અણુ માટે ત્રિજ્યાવર્તી શ્રોડીન્જર સમીકરણ લખો. આયગન મૂલ્યો મેળવવા માટે ત્રિજ્યાવર્તી સમીકરણ ઉકેલો. 7

Write radial schrodinger's equation for hydrogen atom. Solve radial equation for getting eigen values.

અથવા/OR

સમદિકધર્મી દોલક (Isotropic Oscillator) માટે ત્રિજ્યાવર્તી તરંગ સમીકરણ લખો. આ સમીકરણનો ઉપયોગ કરીને સમદિકધર્મી દોલક માટે આયગન મૂલ્ય અને આયગન વિધેય મેળવો.

Write the radial schrodinger equation for isotropic oscillator and solve this equation to derive the eigen values and eigen functions of isotropic oscillator.

(બ) ત્રિ-પારિમાણિક સ્થિતિમાન કૂપ વ્યાખ્યાયિત કરો અને કૂપના અંતર્ગત ભાગ (Interior region)માં ત્રિજ્યાવર્તી શ્રોડીન્જર સમીકરણનો ઉકેલ મેળવો. 7

Define three dimensional square well potential and obtain the solution of radial schrodinger equation in its interior region.

અથવા/OR

સમાન ચુંબકીય ક્ષેત્રમાં વિજભારીત કણ માટેના શક્તિવર્ણપટ અને આયગન વિધેયોની ચર્ચા કરો.

Discuss the energy spectrum and eigen functions for a charged particle in a uniform magnetic field.

4. (અ) (1) અવસ્થા સદિશ (State Vector) નિરૂપણ સમજાવો. 7

Explain the representation of State Vectors.

(2) ગતિકીય ચલો (dynamical variables)નું શ્રેણિક સંકારકો (Matrix Operators)ના સ્વરૂપમાં નિરૂપણ સમજાવો અને દર્શાવો કે

Discuss the representation of a dynamical variables as Matrix operator and show that

$$(x)_A = [F]_A (\Psi)_A$$

અથવા/OR

શ્રોડીન્જર નિરૂપણ (schrodinger representation)ની ચર્ચા કરો અને બતાવો કે

Explain the schrodinger representation and show that

$$\langle x | \hat{P} | \Psi \rangle = -i\bar{h} \frac{\partial \Psi(x)}{\partial x}$$

(બ) યામતંત્રના રેખીય સ્થાનાંતર પ્રેરીત એકેક્ય રૂપાંતરણ માટે દર્શાવો કે $|x\rangle' = e^{-i\varepsilon_j \hat{P}x / \bar{h}} |x\rangle$ 7

Show that $|x\rangle' = e^{-i\varepsilon_j \hat{P}x / \bar{h}} |x\rangle$ for unitary transformation induced by translation of co-ordinate system.

અથવા/OR

યામ સંહતિના પ્રમાણથી પ્રેરીત એકેક્ય રૂપાંતરણ પર સવિસ્તર ચર્ચા કરો. બતાવો કે

Discuss in detail the unitary transformation induced by rotation of co-ordinate system. Show that

$$[\Sigma_x, \Sigma_y] = i \Sigma_z$$

5. ટૂંકમાં જવાબ આપો : 14

Answer in short :

(1) ન્યુમાન વિધેય (Neumann function) લખો.

Write Neumann function.

(2) હેન્કલ વિધેય (Hankel Function)

Write Hankel functions

(i) $H_v^{(1)}(x)$

(ii) $H_v^{(2)}(x)$

(3) બેસેલ વિધેય માટે લંબછેદકીય સંબંધ (Orthogonality relation) લખો.
Write orthogonality relation for Bessel's function.

(4) લીજેન્દ્ર વિકલન સમીકરણ લખો.
Write Legendre differential equation.

(5) વિન્યાસ અવકાશ (configuration space) વ્યાખ્યાયિત કરો.
Define configuaration space.

(6) ઓઈલર-લાગ્રાન્જેનું સમીકરણ લખો.
Write Euler-Lagrange's equation.

(7) હેમિલ્ટનનો સિદ્ધાંત લખો.
State Hamilton's principle.

(8) હોડોગ્રાફ (Hodograph) વ્યાખ્યાયિત કરો.
Define Hodograph.

(9) અવસ્થા વિધેય (state vector) વ્યાખ્યાયિત કરો.
Define state vector.

(10) હિલબર્ટ અવકાશ (Hilbert Space) વ્યાખ્યાયિત કરો.
Define Hilbert Space.

(11) $\hat{x}$ અને $\hat{P}$ વચ્ચેની ક્વોન્ટમ શરત લખો.
Write quantum condition between $\hat{x}$ and $\hat{P}$

(12) ગેજ ટ્રાન્સફોરમેશન (Gauge Transformation) એટલે શું ?
What is Gauge Transformation ?

(13) ગોળીય ધ્રુવીય યામો (Spherical Polar co-ordinates) અને પેરાબોલિક (Parabolic) યામ વચ્ચેનો સંબંધ લખો.
Write relation between Spherical Polar Co-ordinates and Parabolic Co-ordinates.

(14) બીટા ક્ષયની પ્રક્રિયામાં પેરીટીનું સંરક્ષણ થાય છે કે નહી ?
Does parity is conserved in the process of beta decay ?

Seat No. : _____

AC-112

April-2015

B.Sc., Sem. VI

CC-307 : Physics

(Mathematical Physics, Classical Mechanics & Quantum Mechanics)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) Attempt **all** the questions.
(2) **All** questions carry equal marks.
(3) Symbols used have their usual meaning.

1. (a) Prove that $\cos(x) = J_0(x) + 2 \sum_{n=1}^{\infty} (-1)^n J_{2n}(x)$. 7

OR

Prove that $J_{n+3}(x) + J_{n+5}(x) = \frac{(2n+8)}{x} J_{n+4}(x)$.

(b) (i) Prove that $J_n(x) = (-1)^n x^n \left(\frac{1}{x}\frac{d}{dx}\right)^n \left(\frac{\sin x}{x}\right)$. 7

(ii) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

OR

(i) Prove that Legendre polynomial satisfies the following orthogonality relation.

$$\int_{-l}^{+l} P_m(x)\, P_n(x)\, dx = \frac{2}{2n+1} \delta mn.$$

(ii) To calculate Gama function $\gamma(-5/2)$:

2. (a) Obtain Hamiltonian principle from Newton's equation of motion. 7

OR

Distinguish between Hamiltonian formulation and Lagrangian formulation.

(b) Derive Euler Lagrange's equation of motion using δ notation. Discuss it's physical significance. 7

OR

Obtain the Hamiltonian for a change partial moving in an electromagnetic field.

3. (a) Find the Solution of Schrodinger equation in the interior region of a three dimensional square well potential. 7

OR

Write the radial schrodinger equation for Hydrogen atom. Separate this equation in Parabolic co-ordinate.

2.

(b) Starting with differential equation 7

$$\rho \frac{d^2L}{d\rho^2} + [2l + 2 - \rho] \frac{\partial L}{\partial \rho} + [\tau - l - 1] L(\rho) = 0$$

Obtain the normalized radial wave function for Hydrogen atom.

$$\text{Hint : } \int_0^\infty e^{-\rho} . \rho^{p+1} \left[L_q^p(\rho) \right]^2 d\rho = \frac{(2q - p + 1)(q!)^3}{(q - p)!}$$

OR

In case of anisotropic oscillator establish eigen value equation for energy and show that how it lead to energy eigen value E_n.

4. (a) Explain in brief : 7

(i) Hibert space

(ii) Hermition operator

(iii) Unitary operator

OR

Define projection operator. Show that the sum of all the projection operator is 1.

Obtain the relation $\hat{A} = \sum a \ \hat{P}a$.

(b) Prove that $\langle x \setminus \hat{P} \setminus \phi \rangle = -\frac{h}{2\pi i} \frac{\partial \phi(x)}{\partial x}$. 7

OR

Discuss in detail about Space inversion.

5. Answer in short : **14**

(1) Write down Bessel's differential equation.

(2) Write down values of $H_0(\rho)$ and $H_1(\rho)$ for Hermite polynomial.

(3) Define Gama function.

(4) Write down value of $P_3(x)$ for Legendre polynomial.

(5) What is geodesic ?

(6) Write down Lagrangian for L-C-R series circuit.

(7) State Eulers theorem.

(8) Write down Lagrangian for simple pendulum.

(9) Write down equation for Bohr radius.

(10) Write down energy eigen value for n = 0 isotropic oscillator.

(11) Write commute condition between ϕ and L_2.

(12) $[\Sigma_x, \Sigma_y] =$ _____.

(13) Write down the radial wave function for H atom.

(14) Give the definition of Parity.

Seat No. : _____

AB-113

April-2016

B.Sc., Sem.-VI

CC-307 : Physics

(Mathematical Physics, Classical Mechanics & Quantum Mechanics)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) All questions carry equal marks.
(2) Symbols used have their usual meaning.

1. (A) (i) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. 7

(ii) Prove that $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n+1}(x)$

OR

(i) Prove that $N_{\nu-1}(x) + N_{\nu+1}(x) = \frac{2\nu N_\nu(x)}{x}$

(ii) Prove that $N_{\nu-1}(x) - N_{\nu+1}(x) = 2N'_\nu(x)$

(B) Prove that Legendre Polynomial satisfies the following orthogonality condition 7

$$\int_{+l}^{-l} P_m(x)\, P_n(x)\, dx = \frac{2}{2n+1}\delta_{mn}$$

OR

(i) Using the Bessel's function show that
$J_n(-x) = (-1)^n J_n(x)$ where n is integer.

(ii) Show that $xP'_n(x) - P'_{n-1}(x) = nP_n(x)$

2. (A) Explain Geodesis. Show that Geodesis of a spherical surface are great circles. 7

OR

What is Hamilton's principle ? Show that the shortest distance between two point in plane is a straight line.

(B) Using δ notation, obtain Euler-Lagrange's equation of motion. 7

OR

Obtain Hamilton's of system for a simple harmonic pendulum with moving support.

2.

3. (A) What is isotropic oscillator ? Using radial equation, solve the problem of isotropic oscillator. 7

OR

What is anisotropic oscillator ? Obtain normalized radial wave function for anisotropic oscillator.

(B) Discuss the energy spectrum and eigen functions for a charged particle in a uniform magnetic field. 7

OR

Write the radial Schrodinger equation for H-atom. Solve radial equation to obtain energy eigen values.

4. (A) Discuss the Hilbert space of state vector. Prove the orthogonality theorem for eigen vector of a self adjoint operator using Dirac notation. 7

OR

Discuss the representation of a dynamical variables in matrix operator and show that $(x)_A = [F]_A(\psi)_A$.

(B) Explain the projection operator. Show that the sum of all projection operator is 1. Obtain the relation. $\hat{A} = \sum a\hat{P}_a$ 7

OR

Write short note on Time Reversal.

5. Answer in short : 14

(1) Write down Bessel's function $J_n(x)$.

(2) Write down the generating function of Bessel's functions.

(3) Write down value of $P_0(x)$ for Legendre polynomial.

(4) Write down value of $J_{+\frac{1}{2}}(x)$ for Bessel's function.

(5) State Euler Theorem.

(6) Write down Hamiltonian for a charge particle moving in an electromagnetic field.

(7) Write down Lagrangian for L-C-R parallel circuit.

(8) What is phase space ?

(9) Write down complete wave function of H atom ψ_{100} = _____ ?

(10) Ground state energy of anisotropic oscillator is _____.

(11) Define square well potential three dimension

(12) $[\sum x, \sum y]$ = _____ ?

(13) $(A^+)^+$ = _____ ?

(14) $\langle\psi/\phi\rangle^*$ = _____ ?

Seat No. : 134

AG-107

April-2017

B.Sc., Sem.-VI

CC- 307 : Physics

(Mathematical Physics, Classical Mechanics & Quantum Mechanics)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) All questions carry equal marks.

(2) Symbols have their usual meanings.

1. (a) Using, $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$ prove that, 7

(1) $xJ_n' = nJ_n - xJ_{n+1}$

(2) $xJ_n' = -nJ_n + xJ_{n-1}$

OR

Obtain Rodrigue's Formula.

(b) Prove that 7

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x\sin\theta)\, d\theta$$

OR

Prove that,

(1) $xP_n'(x) - P_{n-1}'(x) = nP_n(x)$

(2) $P_n'(x) - xP_{n-1}'(x) = nP_{n-1}(x)$

2. (a) Prove that the shortest distance between two points in a plane is a straight line. 7

OR

State and explain Hamilton's principle. Show that Hamiltonian function is equal to total energy of the conservative system.

(b) If a particle of mass m is moving under the action of gravity on the surface of a smooth sphere of radius *l*, obtain the angle at which the particle flies off from the surface using the Lagrangian method of undetermined multiplier. 7

OR

Obtain Euler's equation of motion using techniques of calculus of variation.

2.

3. (a) What is square well potential ? Using radial Schrodinger equation obtain the solution in the interior region of a three dimensional square potential well. 7

OR

In the outer region of a three-dimensional square well potential, discuss non-localized states ($E > 0$).

(b) Obtain the energy eigen value E_n for isotropic harmonic oscillator in three dimension using radial Schrodinger equation. 7

OR

Using the equation $\rho \frac{d^2L}{d\rho^2} + (2l + 2 - \rho)\frac{dL}{d\rho} + (\lambda - l - 1)\,L = 0$

Obtain the energy eigen value, $E_n = -\frac{\mu z^2 e^4}{2\hbar^2 n^2}$, $n = 1, 2, 3, \ldots..$

for hydrogen atom.

4. (a) Prove for self adjoint operator $\hat{A}$, 7

(1) Its eigen value is always real.

(2) For Eigen states $|a>$ and $|a'>$ of $\hat{A}$, $\langle a'|a\rangle = 0$ when their eigen values are unequal.

OR

Prove that for momentum operator $\hat{P}$, $\langle x|\hat{P}|\psi\rangle = -i\hbar \frac{d\psi(x)}{dx}$

(b) Prove that 7

$|X'> = e^{(-i\theta_n \cdot \hat{L}/\hbar)}\,|X>$

OR

Write a short note on space inversion.

5. Answer in short : 14

(1) Write down Legendre differential equation.

(2) Write spherical Neumann function.

(3) Write down $J_{+\frac{1}{2}}(X)$ for Bessel's function.

(4) Write Hamilton's canonical equations of motion.

(5) Draw phase space diagram of one dimensional oscillator.

(6) Write down Lagrangian for L-C-R series circuit.

(7) Write quantum condition between $\hat{x}$ and $\hat{p}$.

(8) Whether π° meson is a psuedoscalar particle or a psuedovector particle ?

(9) Define linearity of a linear operator $\hat{A}$.

(10) Define projection operator.

(11) Write down the value of $[\Sigma_x, \Sigma_y]$.

(12) Do isolated systems possess translation and rotation invariance ?

(13) Does the parity is conserved in β-decay ?

(14) Write the potential $V(r)$ for H-atom having nuclear charge Ze, electronic charge $-e$ and the distance between electron and nucleus is r.

Seat No. : _____

AB-109

April-2018

B.Sc., Sem.-VI

CC-307 : Physics

(Mathematical Physics, Classical Mechanics and Quantum Mechanics)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are carry equal marks.
(2) Symbols used have their usual meaning.
(3) Figures on R.H.S. show that total mark of questions.

1. (a) Using the Bessel's function show that : 7

(1) $J_n(-x) = (-1)^n J_n(x)$, where n is integer

(2) Prove that $\cos(x) = J_o(x) + 2 \sum_{n=0}^{\infty} (-1)^n J_{2n}(x)$

OR

(1) Prove that $n(x) = (-1)^n x^n \left(\frac{1}{x}\frac{d}{dx}\right)^n \left(\frac{\cos x}{x}\right)$.

(2) Prove that Legendre polynomial satisfies the following orthogonality relation.

$$\int_{-l}^{+l} P_m(x)\, P_n(x)\, dx = \frac{2}{2n+1} \delta mn$$

(b) Prove that $J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x \sin\theta)\, d\theta$ 7

OR

Prove that for $\nu = n$ an integer

(1) $J_{\nu-1}(x) + J_{\nu+1}(x) = \frac{2\nu}{x} J_\nu(x)$

(2) $J_{\nu-1}(x) - J_{\nu+1}(x) = 2J'_\nu(x)$

2. (a) Using δ notation derive Euler Langrang's equation of motion. 7

OR

Obtain the Hamiltonian for a charged partical moving in an electromagnetic field.

(b) Obtain Hamiltonian principle from Newton's equation of motion 7

OR

Obtain Lagrangian for a series LCR and parallel LCR electric circuit on the basis of electromechanical analogies

3. (a) Using the following differential equation of the H atom for L(ρ). 7

$$\rho \cdot \frac{d^2L}{d\rho^2} + (2l + 2 - \rho)\frac{dL}{d\rho} + (\lambda - l - 1)L = 0$$

Prove that $E_n = \frac{\mu e^4}{2\hbar^2 n^2}$ Where n = 1, 2, 3.

OR

For three dimensional isotropic oscillator show that $E_n = \left(n + \frac{3}{2}\right)$ hf. 7

(b) Define three dimensional square well potential and obtain the solution of radial Schrodinger equation in it's interior region.

OR

Discuss the energy spectrum and eigen functions for a changed partical in a uniform magnetic field.

4. (a) Write a short note on Hilbert space. 7

18-19.

OR

Explain the transformation of a dynamical variables and prove that

(i) $\hat{x}' = \hat{x} - \epsilon_L$ (ii) $\hat{p}' = \hat{p}_x + \theta\hat{p}_y$

(b) Discuss in detail about space inversion. 7

OR

Explain the Hermition operator and show that :

(i) $\langle\phi|A = (A^+|\phi)^*$ (ii) $\langle\psi|A^+ = (A|\psi)^*$

5. Answer in short : 14

(1) Write down value of $J_{-\frac{1}{2}}(x)$ for Bessel's function.

(2) Write down the orthogonality relation for Bessel's function.

(3) Write down value of $P_3(x)$ for Legendre polynomial.

(4) Express $f(x) = 8x^3 - 2x + 4$ in terms of Hermite polynomial.

(5) What is geodesic ?

(6) State Euler theorem.

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(7) State Hamiltonian principle.

(8) Define configuration space.

(9) What is Isotropic oscillator ?

(10) The wave function of electron in H atom for n = 1, l = 0 and m = 0 is _____.

(11) Write the Schrodinger equation for H atom in parabolic co-ordinate.

(12) $(A^+)^+$ = _____

(13) $[\Sigma_y, \Sigma_z]$ = _____

(14) Define projection operator.
