Gujarat University

B.Sc. (Sem - III)

December - 2014

201 : Mathematics

(Calculus - I)

[Time : 3 Hours] **[Max. Marks : 70]**

Instructions : **(1)** **All questions are compulsory and carry equal marks.**

(2) **Figures to the right indicate marks of the question/sub-question.**

1\. (a) Define limit of function of two variables. Use this definition to find $\lim\limits_{(x, y) \to (2, 1)} \dfrac{2x + y}{3y - x}$ **(07)**

OR

Let function Q(x) is continuous at a point $(a, Q(a)) = (a, b)$ and $\lim\limits_{(x, y) \to (a, b)} f(x, y)$ exists and is equal to $l \in R$ then prove that $\lim\limits_{x \to a} f(x, Q(x))$ exists and is equal to $l \in R$

(b) Define iterated limits. Find iterated limits of **(07)**

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}; (x, y) \neq (0, 0)$$

$$= 2; \qquad (x, y) = (0, 0) \text{ at point } (0, 0)$$

OR

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Discuss continuity for the function,

$$f(x, y) = \tan^{-1}\left(\frac{y}{x}\right); \quad x \neq 0$$

$$= 0 \qquad ; x = 0 \text{ at point } (0, 0)$$

2. (a) Define directional derivative. If (07)

$$f(x, y) = \frac{xy^3}{x^2 + y^6}; \quad (x, y) \neq (0, 0)$$

$= 0 \quad ; (x, y) = (0, 0)$. Then find directional derivative of function f at point (0, 0) along the direction $\left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{6}}\right)$.

OR

State and prove Young's theorem.

(b) State and prove Schwartz's theorem. (07)

OR

$$\text{If } f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}; \quad (x, y) \neq (0, 0)$$

$$= 0 \qquad ; (x, y) = (0, 0)$$

then find $f_{xy}(0, 0)$; $f_{yx}(0, 0)$; $f_{xx}(0, 0)$; $f_{yy}(0, 0)$

3. (a) State and prove Euler's theorem for homogeneous function. (07)

OR

Find the extreme value of the function.

$$f(x, y) = 2(x - y)^2 - x^4 - y^4$$

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(b) If $u = \operatorname{cosec}^{-1} \sqrt{\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}$ then prove that **(07)**

(1) $$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = \frac{-1}{12} \tan u$$

(2) $$x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 2xy \cdot \frac{\partial^2 u}{\partial x\, \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2} =$$

$$\frac{\tan u}{144} (13 + \tan^2 u)$$

OR

Divide 15 into three parts such that their product shall be maximum.

4. **(a)** Find radius of curvature of a curve **(07)**

$$y = f(x),\ \rho = \frac{(1 + y^{12})^{3/2}}{y^{11}}.$$

OR

Find radius of curvature of a curve

$x = f(t),\ y = g(t).$

(b) Find the radius of curvature of the curve

$x = a \cos^3 \theta;\ y = a \sin^3 \theta$ i.e. $x^{2/3} + y^{2/3} = a^{2/3}$

OR

Find the double point of the curve $x^3 + x^2 - 4y^2 = 0.$

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5. **Answer the following in short :** (14)

(a) Define Partial Derivatives.

(b) If $u = \log(x^2 + y^2)$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

(c) Define continuous function.

(d) Define homogeneous function.

(e) State L'Pittal Rule.

(f) State MacLaurin's theorem

(g) Find radius of curvature of the circle $x^2 + y^2 = 1$.

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GUJARAT UNIVERSITY

November - 2013

B.Sc. SEMESTER - III

MAT - 201 : Mathematics (CC-201) (Advance Calculus - I)

Time : 3 Hours] [Max. Marks : 70

Instruction : All questions are compulsory.

1. **(A) Attempt any two :**

(1) Using definition Evaluate : $\lim_{(x,y)\to(1,2)} (xy - 3x + 4)$

(2) Prove : If $\phi(x)$ is continuous function at point $(a, \phi(a)) = (a, b)$ & $\lim_{(x,y)\to(a,b)} f(x, y) = l \in R$, then $\lim_{x\to a} f(x, \phi(x))$ exist and equal to l.

(3) Discuss the continuity of function $f(x, y)$ at point (0, 0)

$$\text{if } f(x,y) = \left.\begin{array}{ll} \dfrac{x^3 - y^3}{x^2 + y^2} & ; x^2 + y^2 \neq 0 \\ = 0 & ; x^2 + y^2 = 0 \end{array}\right\}$$

(B) Attempt any one :

(1) Evaluate : $\lim_{(x,y)\to(0,0)} \dfrac{\sin(x+y)}{x+y}$

(2) Evaluate the iterated limits of $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ as $(x, y) \to (0, 0)$

2. **(A)** State and prove Young's theorem.

OR

Prove : If for open set, $E \subset R^2$, $f : E \to R$ is defined and if $f_x(x, y)$, $f_y(x, y)$ exist and continuous at point $(x, y) \in E$, then function f is differentiable at point $(x, y) \in E$.

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(B) Attempt any two :

(1) Find the directional derivative of function

$$f(x, y) = \left.\begin{aligned} &\frac{xy^2}{x^2+y^4} && ; x \neq 0,\ y \neq 0 \\ &= 0 && ; x = 0,\ y = 0 \end{aligned}\right\}$$ at point (0, 0) in direction of $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

(2) If U = log $(x^2 + y^2)$ then prove that U is harmonic function of x and y.

(3) Find f_{xx}, f_{yy} and f_{xy} for function

$$f(x, y) = \left.\begin{aligned} &\sin^{-1}\left(\frac{x}{y}\right) && ; y \neq 0 \\ &= 0 && ; y = 0 \end{aligned}\right\}$$

3. (A) State and prove Euler's theorem on homogeneous function.

OR

Prove : If a real valued function f, defined on an open domain $E \subset R^2$, and is differentiable at point $(a, b) \in E$, has an extreme value at (a, b) then $f_x(a, b) = 0$, $f_y(a, b) = 0$.

(B) Attempt any two :

(1) Varify Euler's theorem for the function

$$f(x, y) = x^2 \tan^{-1}\left(\frac{x}{y}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right).$$

(2) If $U = U\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ then show that

$$x^2 \frac{\partial U}{\partial x} + y^2 \frac{\partial U}{\partial y} + z^3 \frac{\partial U}{\partial z} = 0.$$

(3) Find the extreme values of the function
$f(x, y) = 2(x - y)^2 - x^4 - y^4$

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4. (A) Find the radius of curvature of a curve $x = f(t), y = g(t)$. Also find the radius of curvature of a circle $x = r\cos\theta$, $y = r\sin\theta$.

OR

Derive a necessary condition for the existence of a double point on the curve $f(x, y) = 0$.

(B) **Attempt any two :**

(1) Find first three terms in the expansion of $f(x, y) = e^{ax}\cos by$ in powers of x and y.

(2) Find the double point on the curve $x^3 + 3x^2 - y^2 + 3x - 2y = 0$ and explain its type.

(3) Find the radius of curvature of polar curve $r = a(1 + \cos\theta)$

5. (A) **Answer in short (Each of two marks) :**

(1) $$f(x, y) = \left.\begin{array}{ll} \dfrac{x^3 - y^3}{x - y} & ; x - y \neq 0 \\ = 0 & ; x - y = 0 \end{array}\right\}$$ then find $xf_x + yf_y$.

(2) If $U = e^{\frac{xy}{z}}$ then find $\dfrac{\partial^2 U}{\partial x\, \partial y}$.

(3) Find the extreme value of xy under condition $x + y = 1$.

(B) **Answer in short (Each of one mark) :**

(1) Define Connected Set.

(2) Define Iterated limit for function $f(x, y)$ at point (a, b).

(3) Discuss the continuity of $$f(x, y) = \left.\begin{array}{ll} xy + 3 & , (x, y) \neq (1, 2) \\ = 0 & , (x, y) = (1, 2) \end{array}\right\}$$ at point $(1, 2)$.

(4) What is the curvature of curve $ax + by + c = 0$?

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(5) Write the formula to find directional derivative Du f (x).

(6) Define Homogeneous Function.

(7) If the double point is CUSP, then what is relation among r, s & t ?

(8) Write MaClaurin series of function f (x, y).

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