

Seat No. : 640

**MP-128**

**March-2019**

**B.Sc., Sem.-VI**

**310 : Mathematics**

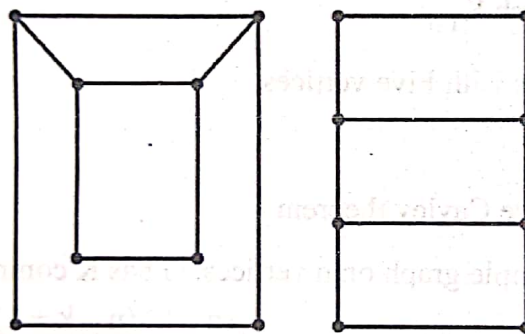
**Time : 2:30 Hours]**

**[Max. Marks : 70**

1. (A) (1) Define the following term with graph : 7

- (i) Adjacent vertices
- (ii) Null graph
- (iii) Edge deleted sub graph
- (iv) k-regular graph

(2) Define isomorphism of a graph. Discuss whether the following graphs are isomorphic or not ? 7



**OR**

(1) For any graph  $G$  with  $e$  edges and  $n$  vertices  $v_1, v_2, v_3, \dots, v_n$  Prove that

$$\sum_{i=1}^n d(v_i) = 2e.$$

(2) Define  $k$ -cube  $Q_k$  and prove that  $Q_k$  has  $2^k$  vertices and  $2^{k-1}$  edges.

(B) Answer in short : (Any TWO)

- (i) What is the smallest positive integer  $n$  such that complete graph  $K_n$  has at least 600 edges.
- (ii) Draw 3 regular graph with 5 vertices.
- (iii) Define neighbourhood set with example.

2. (A) (1) Let  $G$  be acyclic graph with  $n$  vertices and  $k$  connected components, then prove that  $G$  has a  $n-k$  edges. 7

(2) Without drawing actual graph, determine whether the graph is connected or not, whose adjacency matrix is  $A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}$ . 7

OR

(1) If  $T$  is a tree with  $n$  vertices then prove that it has precisely  $n-1$  edges. 7

(2) (i) Let  $u$  and  $v$  be distinct vertices of a tree  $T$ . Then there is precisely one path from  $u$  to  $v$ .

(ii) Let  $G$  be graph without any loops. If for every pair of distinct vertices  $u$  and  $v$  of  $G$  there is precisely one path from  $u$  to  $v$ , then  $G$  is a tree. 7

(B) Answer in short : (Any TWO) 4

(i) Define forest with graph.

(ii) Draw star graph  $K_{1,6}$

(iii) Give two trees with Five vertices.

3. (A) (1) State and prove Cayley theorem. 7

(2) Let  $G$  be a simple graph on  $n$  vertices.  $G$  has  $K$  components then the number of edges of  $G$  satisfies  $n - k \leq m \leq \frac{(n-k)(n-k+1)}{2}$ . 6

OR

(1) Let  $G$  be a graph with  $n$  vertices, where  $n \geq 2$ . Then  $G$  has atleast two vertices which are not cut vertex. 7

(2) Prove that any simple graph with  $n$  vertices and more than  $\frac{(n-1)(n-2)}{2}$ . 6

(B) Answer in short : (Any TWO) 4

(i) Draw Petersen graph.

(ii) Define Cut – Vertex with example.

(iii) Let  $G$  be a connected graph with 14 edges then what is the maximum possible number of vertices in  $G$ ?

4. (A) (1) A connected graph  $G$  is Euler if and only if the degree of every vertex is even. 7
- (2) Prove that the vertex connectivity  $k(G)$  of graph  $G$  is always less than or equal to the Edge connectivity  $\lambda(G)$ . 6

OR

- (1) If  $G$  is a simple graph with  $n \geq 3$  vertices and if  $\deg V + \deg W \geq n$  for each pair of non-adjacent vertices  $V$  and  $W$  then  $G$  is Hamiltonian. 7
- (2) Discuss The Konigsberg bridges problem. 6
- (B) Answer in short : (Any TWO) 4
- (i) How many different Hamiltonian cycle for complete graph  $K_5$ .
- (ii) Define Hamiltonian path and Hamiltonian cycle.
- (iii) Define Closure of graph  $G$ .
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