Seat No.: 640

MN-140

March-2019

B.Sc., Sem.-VI

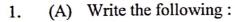
CC-308: Mathematics (Analysis-II)

Time: 2:30 Hours]

[Max. Marks: 70

Instructions:

- (1) All the four questions are compulsory.
- (2) Each of the questions Q.1 and Q.2 are of 18 marks and Q.3, Q.4 are of 17 marks.



- (i) Define Riemann integrable function on [a, b]. Prove: If f is a monotone function on [a, b] then f is Riemann integrable on [a, b]. Show that fg ∈ R [a, b] whenever f, g ∈ R [a, b].
- (ii) If $P = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots, \frac{n}{n}\right\}$ is a partition of [0, 1] and $f(x) = \frac{x}{2}$ then find $\lim_{n \to \infty} U_p(f) \lim_{n \to \infty} L_p(f)$.

OR

(i) State and prove First Mean Value Theorem of Integral Calculus. Verify it for the function f(x) = x + 1 on [0, 1].

(ii) Prove that
$$\frac{\pi^3}{3b} \le \int_{0}^{\pi} \frac{x^2 dx}{a \cos^2 x + b \sin^2 x} \le \frac{\pi^3}{3a}$$
 where, $0 < a < b$.

- (B) Attempt any two out of three in short :
 - (i) State the fundamental Theorem of Calculus.
 - (ii) Evaluate : $\int_{-2}^{2} x|x| dx.$
 - (iii) Show that the constant function is Riemann integrable.

P.T.O.

- 2. (A) Write the following:
 - (i) Define conditional convergence of the series. If (a_n) is a decreasing sequence of positive terms converging to zero, then prove that the series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges. Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$.
 - (ii) If $\sum_{n=1}^{\infty} a_n$ is a convergent series of real numbers then show that

 $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$ is also convergent. Discuss the absolute convergence of the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{\sqrt{n^2+2}-n}{\sqrt{n}} \right).$$

OR

- (i) State and prove Cauchy's condensation test and hence establish p-test $\Sigma \frac{1}{n^p}$ for the series.
- (ii) If $\sum_{n=1}^{\infty} a_n$ diverges and all $a_n > 0$ then show that the series $\sum \frac{a_n}{1 + na_n}$ is divergent.
- (B) Attempt any two out of three in short:
 - (i) Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$
 - (ii) The series $\sum_{n=0}^{\infty} \frac{1}{2^{2n}}$ converges to the sum _____.
 - (iii) If (a_n) is a decreasing sequence of positive terms and Σa_n is convergent then $\lim_{n\to 0} na_n = 0$. Give suitable name to this statement.

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- 3. (A) Write the following:
 - (i) If the series Σa_n is absolutely convergent then prove that any rearrangement of Σa_n has the same sum.
 - (ii) Discuss the convergence of the following power series stating interval of convergence:
 - $(1) \quad \sum_{n=1}^{\infty} \frac{x^n}{4^n n^2}$
 - (2) $\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$
 - (3) $\sum_{n=0}^{\infty} \frac{2^n x^n}{3^n + 4}$

OR

- (i) Prove: If f is defined on $[a, \infty)$ and $\int_{a}^{\infty} |f|$ converges then $\int_{a}^{\infty} f$ converges, and $\left|\int_{a}^{\infty} f\right| \leq \int_{a}^{\infty} |f|$.
- (ii) Examine the convergence of the following improper integrals:

(a)
$$\int_{2}^{\infty} \frac{1}{x(x-1)} \, \mathrm{d}x$$

(b)
$$\int_{1}^{\infty} \frac{\cos x}{x^3} \, \mathrm{d}x$$

(c)
$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} \, \mathrm{d}x$$

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- Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^3 x^n}{4^n}$. (i)
- Find the Cauchy product of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ with itself.
- (iii) Evaluate: $\int x|x|dx$
- (A) Write the following: 4.

State Taylor's theorem. Using Lagrangian form of the remainder show that $\log (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$. For $-1 < x \le 1$

State and prove Binomial series theorem.

- Show that $(1+x)^a \approx 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha-1)(\alpha-2)....(\alpha-n+1)}{n!} x^n$ converges (i) and find its radius of convergence. Give suitable name to this result.
- Obtain in power series solution of the differential equation (1 x) y' + 2y =(ii) 0 with the initial condition y(0) = 2.
- Attempt any two out of three in short:

Obtain the Maclaurin series expansion for $\log (1 + x)$. (i)

- If $y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$ and y(x) = y'(x) then obtain the relation (ii) between the coefficients.
- Examine the validity of the statement $\ln 2 = 1 \frac{1}{2} + \frac{1}{3} \frac{1}{4} + \dots$