Seat No. : 4408

NO-105

November-2017

B.Sc., Sem.-V

SE-305 : Mathematics

(Number Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instruction : All questions are compulsory.

1. (a) State and prove the Division Algorithm theorem. Prove that for $n \geq 1$, $6|n(n+1)(2n+1)$. **9**

OR

Show that for any two non zero integers a & b there exist integers x & y such that $\gcd(a, b) = ax + by$. Express the gcd of 56 and 72 as a linear combination of it.

(b) State and prove the necessary and sufficient condition for linear Diophantine equation has solution. Find the solution of Diophantine equation $14x + 35y = 52$ if exists. **9**

OR

Establish the result :

(i) If a and b are odd integers then $8|a^2 - b^2$

(ii) Product of four consecutive integers is divisible by 24.

2. (a) Prove that there are infinitely many primes of the form $4k + 3$. **9**

Is for any $n \in \mathbb{N}$, $n^2 + n + 17$ prime ? Justify your answer.

OR

(i) If $n > 1$, then show that n! is never a perfect square.

(ii) Find the positive integer n such that $n! + (n+1)! + (n+2)!$ is a perfect square.

(b) (i) Prove : $27 | 2^{5n+1} + 5^{n+2}, \forall n \geq 1$. **9**

(ii) What is the remainder when $\sum_{n=1}^{100} n^5$ is divided by 4.

OR

(i) Solve : $17x \equiv 9 \pmod{276}$

(ii) Find x & y : if 495 | 273x49y5

3\. (a) State and prove the Fermat's theorem. Is converse true ? Justify. **9**

OR

Prove : the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$ has solution iff $p \equiv 1 \pmod{4}$ for any odd prime p. Also find the solution of congruence : $x^2 + 1 \equiv 0 \pmod{31}$

(b) (i) State the Euler's Theorem, and using it prove that : $a^{37} \equiv a \pmod{1729}$ $a \in \mathbb{Z}$.

(ii) Find the unit digit of 7^{100}. **9**

OR

(i) Define : *pseudoprime* & *absolute pseudoprime*, show that 561 is an *absolute pseudoprime*

(ii) Find the last two digits of the number 9^{9^9}.

4\. Give the answer of following in short : (any **eight**) **16**

(1) State the Well Ordering Principle.

(2) State the fundamental theorem of arithmetic. What is the canonical form of 2017 ?

(3) State the Wiloson's theorem.

(4) Define : the complete set of residues modulo n.

(5) Define : Square free integer with illustration.

(6) Define Euler phi function and evaluate : $\phi(2112)$.

(7) Find a prime of the form $n^3 - 1$.

(8) Find r : $\sum_{n=1}^{25} (n!) \equiv r \pmod{9}$

(9) Find all prime numbers that divides 100!.

(10) 50! end with how many zero ?
