Seat No. : _____

NN-106

November-2017

B.Sc., Sem.-V

CC-304 : Mathematics

(Mathematical Programming)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (i) **All** the questions are compulsory and carry **14** marks.

(ii) Notations are usual, where not mentioned.

1. (a) Prove that $K \subset R^n$ is a convex set if and only if every convex linear combination of elements in K also belongs to K. **7**

OR

Prove that the intersection of two convex sets is a convex set.

(b) Determine convexity of the sets :

$S_1 = \{x \in E^n / \| x \| \leq 2\}$ and $S_2 = \{ x \in E^n / \| x \| \geq 1\}$. **7**

OR

A person requires 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C respectively per carton. If the liquid product sells for ₹ 30 per jar and the dry product sells for ₹ 35 per carton, how many of each should be purchased in order to minimize the cost and meet the requirements ?

Formulate the Linear Programming Problem.

2. (a) Describe the usual simplex method for solving a Linear Programming Problem. 7

OR

Describe the Two-Phase Simplex Method for solving a Linear Programming Problem.

(b) Solve the following LPP by big-M method : 7

Maximize $Z = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$ and $x_1, x_2, x_3 \geq 0$.

OR

Solve the following Integer Programming Problem by the Gomory's Cutting plane method :

Maximize $Z = 4x_1 + 3x_2$

Subject to $x_1 + 2x_2 \leq 4$

$2x_1 + x_2 \leq 6$

$x_1, x_2 \geq 0$ and are integers.

3. (a) Explain the concept of duality and principle of duality. 7

OR

By the matrix form of a linear programming problem prove that the dual of the dual is the primal of the Linear Programming Problem :

Also verify it for the following Linear Programming Problem :

Maximize $Z = 4x_1 + 5x_2 + 6x_3$

Subject to $x_1 - 2x_2 - 3x_3 \leq 10$

$3x_1 - 4x_2 + 5x_3 \leq 20$

and $x_1, x_2, x_3 \geq 0$.

(b) Describe the solution of the following LP Problem by solving its dual : 7

Maximize $Z = 4x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 2$ and $x_1, x_2 \geq 0$.

OR

Use the Dual Simplex Method to solve the following LP Problem :

Minimize $Z = 2x_1 + x_2 + x_3$

Subject to $4x_1 + 6x_2 + 3x_3 \leq 8$

$-x_1 + 9x_2 - x_3 \geq 3$

$-2x_1 - 3x_2 + 5x_3 \leq -4$ and $x_1, x_2, x_3 \geq 0$.

4. (a) Explain balanced and unbalanced transportation problems by mathematical formulations. Give brief idea for how to solve an unbalanced transportation problem. 7

OR

What is an assignment problem ? Explain how is it a special case of the transportation problem. Also describe the main differences between them.

(b) Use the Least Cost Method and the Vogel's Approximation method to find initial basic feasible solutions of the following transportation problem : 7

Origins	Destinations				Supply
	D_1	D_2	D_3	D_4	
O_1	15	10	17	18	2
O_2	16	13	12	13	6
O_3	12	17	20	11	7
Demand	3	3	4	5	

OR

Solve the following Assignment problem by the criterion of minimization :

Jobs	Workers			
	A	B	C	D
I	40	33	26	19
II	28	23	18	13
III	28	23	18	13
IV	22	18	14	10

5. Answer any **Seven** of the followings in short : **14**

(1) Define a convex set.

(2) Define an extreme point of a convex set.

(3) Define an optimum solution of a Linear Programming Problem.

(4) Name the methods of solving an LP Problem having an artificial variable.

(5) Explain standard primal form of a Linear Programming Problem.

(6) Define an Integer Programming Problem.

(7) Define a loop in a transportation problem.

(8) Name the method of finding an optimum solution of a transportation problem.

(9) Name any one method of solving an Assignment Problem.
