Seat No. : 4408

# NM-111

**November-2017**

**B.Sc., Sem.-V**

**CC-303 : Mathematics**

**(Complex Variables and Fourier Series)**

**Time : 3 Hours]** **[Max. Marks : 70**

**Instructions :** (1) All the questions are compulsory.
(2) Each question carry **14** marks.

1. (a) State and prove De Moivre's theorem. Hence solve the equation $x^3 + 1 = 0$. 7

**OR**

Define modulus of complex number $z = x + iy$.
If $z_1$ and $z_2$ are complex numbers, then prove that $|z_1| - |z_2| \leq |z_1 - z_2| \leq |z_1| + |z_2|$

(b) Prove that (i) $\sin ix = i \sinh x$, (ii) $\cos ix = \cosh x$ and hence separate $\tanh (x + iy)$ into its real and imaginary parts. 7

**OR**

For what values of z does the series $\Sigma \frac{1}{(z^2 + 1)^n}$ converge ? Also find its sum.

2. (a) Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin. 7

**OR**

Prove that the necessary conditions for a function $f(z) = u + iv$ to be analytic at all points in a region R are (i) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ (ii) $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

(b) If $f(z) = u(r, \theta) + iv(r, \theta)$ be an analytic function and $u = -r^3 \sin 3\theta$, then find a function $v(r, \theta)$ and also express the function $f(z)$ in terms of z. 7

**OR**

Show that the real and imaginary parts of the function $w = \log z$ satisfy Cauchy-Riemann equations $(z \neq 0)$. Also find its derivative.

3. (a) Define conformal transformation. 7
If $f(z)$ is analytic and $f'(z) \neq 0$ at each point z of the domain, then prove that the mapping $w = f(z)$ is conformal.

**OR**

Show that the transformation $w = \frac{5 - 4z}{4z - 2}$ transform the circle $|z| = 1$ into a circle of radius unity in w-plane and find the centre of the circle.

(b) Considering the map $w = ze^{i\frac{\pi}{4}}\sqrt{2}$, determine the region R' of w – plane corresponding to the rectangular region R bounded by the lines $x = 0$, $y = 0$, $x = 2$, $y = 3$ in z-plane. **7**

**OR**

Determine the region R' in w-plane of the infinite strip R bounded by $\frac{1}{4} < y < \frac{1}{2}$ under the mapping $w = \frac{1}{z}$.

4. (a) State and prove Bessel's inequality of the Fourier series. **7**

**OR**

If $\Phi(x)$ is Riemann integrable in $a \le x \le b$ and if $A_n = \int_a^b \Phi(x) \cos nx \, dx$ and $B_n = \int_a^b \Phi(x) \sin nx \, dx$, then prove that $A_n \to 0$, $B_n \to 0$ as $n \to \infty$.

(b) Find the Fourier series expansion for the function $f(x) = x \sin x$ in $[-\pi, \pi]$. Hence deduce that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots$ **7**

**OR**

Find the Fourier series expansion for the function $f(x) = \frac{1}{2}(\pi - x)$ in $(0, 2\pi)$ with period $2\pi$. Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots\ldots$

5. Answer the following in short (any **seven**) : **14**

(1) Find the modulus and principal argument of the complex number $\sqrt{\frac{1+i}{1-i}}$.

(2) Find the square root of the complex number $3 + 4\sqrt{7}i$.

(3) Write C-R equations in polar form.

(4) Is the function $f(z) = e^z$ analytic ? If yes, then find its derivative.

(5) The Fourier series of the even function contains only cosine terms and that of odd function contains only sine terms. Is it true ? Justify.

(6) Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$.

(7) Define Isogonal transformations.

(8) If $(x + iy)^{1/3} = a + ib$, then show that $4(a^2 - b^2) = \frac{x}{a} + \frac{y}{b}$.

(9) Show that $\cosh^{-1}\sqrt{1 + x^2} = \sinh^{-1}x$.