Seat No. : 4408

NL-111

November-2017

B.Sc., Sem.-V

CC-302 : Mathematics

(Analysis – I)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory and carry **14** marks.
(2) Notations are usual.

1. (a) Prove that the set of all rational numbers is countable. **7**

OR

Prove that countable union of countable sets is again countable.

(b) State and prove rational density theorem. **7**

OR

Prove that $\sqrt{11}$ is not rational.

2. (a) If $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\dots + \frac{1}{n!}$ then prove that $2 < \lim\limits_{n \to \infty} S_n < 3$. **7**

OR

If $\{x_n\}$ is a Cauchy sequence of real numbers then prove that $\{x_n\}$ is convergent.

(b) If $S_1 = \sqrt{2}$ and $S_{n+1} = \sqrt{2S_n}$ for every n, then show that (S_n) is monotonic increasing bounded above and $\lim\limits_{n \to \infty} S_n = 2$. **7**

OR

Prove that a bounded and monotonically increasing sequence converges.

3. (a) If $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} g(x) = M \neq 0$, then prove that $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$. **7**

OR

Prove :

(i) If $\lim\limits_{x \to a} g(x) = M \neq 0$ then $\lim\limits_{x \to a} \frac{1}{g(x)} = \frac{1}{M}$

(ii) $\lim\limits_{x \to 0} \frac{1}{x^2} = +\infty$

(b) State and prove intermediate value theorem. **7**

OR

If fa^n f is continuous at "a" and fa^n g is continuous at f(a) then gof is continuous at "a".

4. (a) State and prove Mean Value Theorem. **7**

OR

State and prove the Roll's theorem and verify it for $f(x) = x^3 - 3x + 2$ in $[-1, 2]$

(b) Evaluate **7**

(i) $\lim\limits_{x \to 0} \dfrac{e^x - 2 - x - \frac{x^2}{3}}{\sin^3 x}$

(ii) $\lim\limits_{x \to 0} \left(\dfrac{4}{x \tan x} - \dfrac{1}{x \sin x}\right)$

OR

Suppose that fa^n f is continuous and one-to-one on [a, b] and is differentiable with $f'(x_0) \neq 0$, then prove that f^{-1} is also differentiable at $y_0 = f'(x_0)$ and $(f^{-1})'\ (y^0) = \dfrac{1}{(f'\ f^{-1}(y_0))}$.

5. Answer the following in short : (any **seven**) **14**

(1) In usual notations prove that $[0, 1] \sim (0, 1)$.

(2) If $f(0) = f'(0) = 1$, then evaluate $\lim\limits_{x \to 0} \dfrac{f(x) - 1}{x}$.

(3) Show that R is uncountable.

(4) Define injective and subjective functions with example.

(5) State Cauchy's mean value theorem.

(6) Find $\lim\limits_{x \to -\infty} \dfrac{|x-2|}{x}$ and $\lim\limits_{x \to +\infty} \dfrac{|x-2|}{x}$.

(7) Give an example of a sequence which is bounded but not convergent.

(8) State only L" Hospital's Rule.

(9) Prove that every convergent sequence is Cauchy seq. too.