Seat No. : 6639

NE-116

November-2018

B.Sc., Sem.-V

CC-301 : Mathematics

(Linear Algebra – II)
(Theory)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Write the question number in your answer book as shown in the question paper.

(3) The figure to the right indicates marks of the question.

1. (a) (i) Let V be an n-dimensional vector space and let B = $\{x_1, x_2, \dots x_n\}$ be a basis of V. Then prove that there is a uniquely determined basis $B^* = \{f_1, f_2, f_3, \dots f_n\}$ of V^* such that $f_i(x_j) = \delta_{ij}$ i, j = 1, 2, 3, …, n. **7**

(ii) Let $T : V_5 \to V_3$ be a linear map defined by $T(e_1) = \frac{1}{2} f_1$, $T(e_2) = \frac{1}{2} f_1$, $T(e_3) = f_2$, $T(e_4) = f_2$, $T(e_5) = 0$, where $\{e_1, e_2, e_3, e_4, e_5\}$ is the standard basis for V_5 and $\{f_1, f_2, f_3\}$ is the standard basis for V_3. Then solve the operator equation $T(x_1, x_2, x_3, x_4, x_5) = (1,1,0)$. **7**

OR

(i) Prove that the annihilators W^0 is a subspace of dual space V^*. **7**

(ii) Find the dual basis of the basis B = {(1, 1, 1), (1, 0, –1), (0, 3, 4)} of the vector space V_3. **7**

(b) Give the answer in brief. (any **two**) 4

(i) Let $T : V_3 \to V_2$ and $S : V_3 \to V_2$ be two linear maps define by

$T(x_1, x_2, x_3) = (x_1 - x_2, x_2 - x_3)$ and $S(x_1, x_2, x_3) = (2x_1, x_2 - x_3)$ then find $(S + T)$.

(ii) Define : Operator equations.

(iii) Is the commutative law always satisfied for the product of two linear maps ? Justify your answer.

2. (a) (i) Let W be a vector subspace of an inner product space V and $W^{\perp}$ be the orthogonal complement of W, then prove that $V = W \oplus W^{\perp}$. 7

(ii) Prove that the sum of the squares of the diagonals of a parallelogram equals the sum of the squares of its sides. 7

OR

(i) If $(V, <, >)$ is an inner product space, then prove that a linear map $T: V \to V$ is orthogonal linear map if and only if $\|T(x)\| = \|x\|$ for all $x \in V$. 7

(ii) Apply Gram-Schmidt orthogonalization process to the basis $B = \{(-1, 0, 1), (1, -1, 0), (0, 0, 1)\}$ in order to get the orthonormal basis for R^3. 7

(b) Give the answer in brief : (any **two**) 4

(i) Define : Orthonormal Basis.

(ii) State Cauchy – Schwartz's inequality.

(iii) Compute the angle between the vector (1, 0) and (0, 1).

3. (a) (i) State and prove Cramer's rule. **7**

(ii) If $A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 3 & 0 & 3 & 1 \\ 1 & 6 & 4 & 0 \\ 4 & -4 & 0 & 2 \end{bmatrix}$ then find det A by using the Laplace Expansion about the first row of the matrix A. **6**

OR

(i) Prove that det (AB) = det (A) det (B) and det $(A^{-1}) = \det(A)^{-1}$. **7**

(ii) Using Cramer's Rule, solve the system of equations :

$x + y + z = 2,\ 2x + 3y - z = 1,\ x + y + 3z = -5.$ **6**

(b) Give the answer in brief. (any **two**) **4**

(i) If $x = y$ and x & y are column vector of vector space V_2, then find the value of det (x, y).

(ii) Find the det A if $A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$.

(iii) If $A \in M(n, R)$ and the column vectors $(A_1, A_2, A_3, \ldots A_n)$ of A is linearly independent, then find the value of det (A).

4. (a) (i) Let $T : V \to V$ be a symmetric linear map on a finite dimensional inner product space, then prove that there exist an orthonormal basis of V consisting of eigen vector of T. **7**

(ii) Diagonalize the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$. **6**

OR

(i) Let $T : V \to V$ be symmetric linear map, then prove that the eigen vector v_i with eigen values λ_i, $i = 1, 2$ with $\lambda_1 \neq \lambda_2$ are orthogonal each other. **7**

(ii) Find A^{-1} of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ using Cayley – Hamilton Theorem. **6**

(b) Give the answer in brief. (any **two**) **4**

(i) Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$.

(ii) State Cayley – Hamilton theorem.

(iii) Write an equation of Hyperboloid of one sheet.
