Seat No. : 4408

NK-106

November-2017

B.Sc., Sem.-V

CC-301 : Mathematics

(Linear Algebra – II) Theory

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Write the question number in your answer-book as shown in the question paper.

(3) The figure to the right indicate marks of the question.

1. (a) Let V be a finite dimensional vector space over the field F and let W be a subspace of V and W^0 is inihilator of W. Then prove that $\dim W + \dim W^0 = \dim V$. **7**

OR

Let V be an n-dimensional vector space and let $B = \{x_1, x_2, \ldots x_n\}$ be a basis of V. Then prove that there is a uniquely determined dual basis $B^* = \{f_1, f_2, f_3, \ldots, f_n\}$ of dual space V^* such that $f_i(x_j) = \delta_{ij}$ $i, j = 1, 2, 3, \ldots, n$.

(b) Let V be the vector space of polynomials p(t) over real number R. Define a map $I : V \to R$ is given by $I(p(t)) = \int_0^1 p(t)dt$, then show that the map $I : V \to R$ is linear functional on V. **7**

OR

Find the dual basis of the basis $B = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ of the vector space V_3.

2. (a) State and prove Cauchy-Schwartz's inequality. **7**

OR

Prove that a finite dimensional inner product space V has an orthonormal basis.

(b) Let V be vector space and let any $x, y \in V$, then prove that $4\langle x, y\rangle = \|x + y\|^2 - \|x - y\|^2$. **7**

OR

If V be a vector space of polynomials in t with inner product given by

$$\langle p(t), q(t)\rangle = \int_0^1 P(t) \cdot q(t)\, dt$$ then find $\langle p(t), q(t)\rangle$ and $\|p(t)\|$.

Here $P(t) = t + 2$ and $q(t) = t^2 - 2t - 3$.

3. (a) State and prove Cramer's rule. **7**

OR

State and prove Laplace expansion.

(b) Use the Cramer's rule to solve : $2x - y + z = 3$; $x + 3y - 2z = 1$; $x - y + 3z = 4$ **7**

OR

Compute the det A without expansion if $A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 0 & 4 & 1 \\ 1 & 6 & 5 & 0 \\ 4 & -3 & 0 & 1 \end{bmatrix}$

4. (a) State and prove the Cayley-Hamilton's theorem. **7**

OR

Prove that all characteristic roots of a symmetric and real matrix are real.

(b) Let the matrix $A = \begin{bmatrix} 1 & 4 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Find inverse of matrix A using Cayley-Hamilton Theorem. **7**

OR

Identify the quadratic equation $x^2 + y^2 + z^2 + 4xy + 4yz - 4zx = 27$.

5. Answer the following questions in short : (any **seven**) **14**

(1) Define : Operator equations.

(2) Define : Dual basis.

(3) Define : Inner product space.

(4) Find the angle between the vectors (1, 0) and (1, 1).

(5) Find the value of x such that $\begin{vmatrix} 3-x & 2 \\ 2 & 8-x \end{vmatrix} = 0$.

(6) If matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and matrix $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, then show that $\det A + \det B \neq \det(A + B)$.

(7) Find the eigen value of the matrix $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$.

(8) Write matrix form of the conic $10x^2 + 2xy + 7y^2 = 100$.

(9) Show that the vector (2, 3) and (–3, 2) are orthogonal to each other.