Seat No. : 2041

MB-114

November-2016

B.Sc., Sem.-V

CC-301 : Mathematics

(Linear Algebra-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) **All** the questions carry **14** marks.

(3) Right hand side figures indicate marks of the question/sub question.

(4) Notations are usual.

1. (a) If $T : U \to V$ is linear map, $v_0 \in R(T)$ and if $T(u) = \overline{0}_V$ has a nontrivial solution $u \neq \overline{0}_U$ then prove that the operator equation $T(u) = v_0$ has an infinite number of solutions. **7**

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OR

State and prove the dual basis existence theorem.

(b) If a linear map $T : V_3 \to V_3$ is defined as $T(e_1) = e_1 + e_2$, $T(e_2) = e_2 + e_3$, $T(e_3) = e_3 + e_1$, then solve the operator equation $T(x_1, x_2, x_3) = (4, 6, 8)$. **7**

OR

Find the dual basis of the basis $B = \{(1, 2, 3), (3, 1, 2), (2, 3, 1)\}$ for the vector space V_3.

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2. (a) Prove that a finite dimensional inner product space has an orthogonal basis. **7**

OR

Define an orthogonal linear map.

If $(V, <, >)$ is an inner product space then prove that a linear map $T : V \to V$ is an orthogonal linear map if and only if $\| T(x) \| = \|x\|$ for all $x \in V$.

(b) If for $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$ the map $<, >$ is defined as $<x, y> = y_1 [x_1 - x_2] + y_2[2x_2 - x_1]$ then show that $<,>$ is an inner product on R^2. 7

OR

Apply the Gram-Schmidt orthogonalization process to the basis
$B = \{(0, -1, 1), (1, 0, -1), (1, 1, 0)\}$ in order to get orthonormal basis for V_3.

3. (a) If $i \neq j$, $\alpha \in R$ and if $\det : V^n \to R$ is a function satisfying the expected properties of the determinant then prove the followings : 7

(i) $\det(v_1, v_2, \ldots, v_i, \ldots, v_j, \ldots, v_n) = \det(v_1, v_2, \ldots, v_i + \alpha v_j, \ldots, v_j, \ldots, v_n)$

(ii) $\det(v_1, v_2, \ldots, v_i, \ldots, v_j, \ldots, v_n) = -\det(v_1, v_2, \ldots, v_j \ldots, v_i, \ldots, v_n)$

OR

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State and prove the Cramer's rule for solving a system of linear equations.

(b) If $A = \begin{pmatrix} 1 & 0 & 3 & 4 \\ 0 & -1 & 4 & 5 \\ 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & -1 \end{pmatrix}$ then find det A by applying the Laplace Expansion about the last row of the matrix A. 7

OR

If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & -1 \\ 2 & 0 & 4 & 2 \\ 7 & 3 & 1 & -1 \end{pmatrix}$ then compute det A without expansion.

4. (a) Express the characteristic equation of 2×2 matrix A in terms of Trace of A and det A. Also prove that a 2×2 real and symmetric matrix has only real eigen values. 7

OR

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State and prove the Cayley-Hamilton's theorem.

(b) Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, also find the modal matrix which diagonalizes A. 7

OR

Identify the quadric in R^3 given by
$f(x, y, z) \equiv 4xz + 4y^2 + 8y + 8 = 0$

5. Answer any **seven** of the following questions in short : **14**

(a) Define addition and composition of linear maps.

(b) Define a linear functional and give one example of it.

(c) Define an endomorphism and an isomorphism.

(d) Define and illustrate an inner product space.

(e) Define orthogonal set and give one example of it.

(f) State the Laplace Expansion.

(g) Find det A if $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 5 & 4 & 3 \end{bmatrix}$

(h) Define eigen value and eigen vector of an endomorphism.

(i) Define a bilinear map and a quadric.